experiment

Random Experiment (TRIAL) :

An experiment in which the result is uncertain is called a random experiment.

Ex: Tossing of a coin

Drawing a card from a pack of cards.

Throwing cubical die

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Sample Space :

The set of all possible outcomes in a random experiment is called a sample space

Ex: 1) when a coin is tossed

$S = \{H, T\}$

2) when two coins are tossed

$S = \{HH, HT, TH, TT\}$

3) when a cubical die is thrown

$S = \{1, 2, 3, 4, 5, 6\}$

4) when two cubical dice are thrown mutually

$S = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6),$
$(2,1)\ \ldots\ \ldots\ (2,6)$
$\vdots$
$(6,1)\ \ldots\ \ldots\ (6,6)\}$

EVENT :

one or more outcomes of a random experiment is called an event.

Ex:
1) when a coin is tossed

$S = \{H, T\}$

$E_1 = \{H\}$ getting a head is an event.

2) when a cubical die is thrown

$S = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{1, 3, 5\}$ getting an odd number is an event

$E_2 = \{2, 4, 6\}$ getting an even number is an event.

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MUTUALLY EXCLUSIVE EVENTS

Two events are said to be mutually exclusive events, if the two events do not happen together at the same time.

Ex:
1) when a coin is Tossed

$S = \{H, T\}$

Head & Tail are mutually exclusive

Ex: 2) when a cubical die is Thrown

$S = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{1, 3, 5\}$ / approach

likely

$E_2 = \{2, 4, 6\}$

E_1 and E_2 are mutually exclusive

likely

PROBABILITY

The word probable is used very commonly in everyday Language to mean something which is very likely to happen. The concept of probability is to quantify (get) the uncertainity. (result of any Particular Experiment) The measure of chance is denoted as probability. ie The probability of a given event is an expression of likelihood or chance or occurence of an event.

A probability is a number which ranges from 0 to 1. The event with zero propability represents "absolute impossibility". The event with probability 1 represents absolute certainity. The other values between 0 and 1 would indicate the relative chance of occurence of an event.

Certainty

Three approaches to Probability

1. Classical or priori probability
2. Relative frequency approach
3. Subjective approach

originate

CLASSICAL APPROACH TO PROBABILITY

The classical approach to probability is the oldest and simplest. It originated in problems related to games of chance such as throwing of coins, dice etc.

Let 'n' be the total number of outcomes in a random experiment, let 'm' be the number of outcomes favourable to the event 'E', Then probability of the event E is given by $P(E) = \frac{m}{n}$

$$P(E) = \frac{\text{no. of outcomes favourable to the event E}}{\text{Total no. of outcomes}}$$

<u>Ex:</u> when a card is drawn from a pack of cards, the probability of getting a King card is $P(K) = \frac{4}{52}$

Relative frequency Approach

The classical definition is difficult or impossible to apply as soon as we deviate from the field of Coins, dice and Cards.

Let 'n' be the number of times a random experiment repeated, let 'm' be the number of times an event 'E' occured Then Probability of the event E is given by

$$P(E) = \frac{m}{n}$$

$$P(E) = \frac{\text{no. of times an event E occured}}{\text{Total no. of times a random experiment repeat}}$$

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Ex: A person hits a target 3 times, out of 10 attempts. Then probability of this hitting the Target is $\frac{3}{10}$

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incidence

Subjective approach

The subjective probability is defined as the probability assigned to an event by an individual based on whatever incidence is available. Such probabilities are based on the belief of the person making the statement. It is not constant. It varies from person to person.

assignment

AXIOMS OF PROBABILITY (or) Axiomatic approach to probability

In this approach no precise definition of probability is given. The whole field of probability theory for finite samples is based upon the following axioms

1. The probability of an event ranges from zero to one. If the event cannot take place its probability shall be zero. And if it is certain its probability shall be 1

$$0 \leq P(E) \leq 1$$

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(2) The probability of entire sample space is 1.

$$P(S) = 1$$

(3) If A and B are mutually exclusive events then the probability of occurence of either A or B is denoted by $P(A \cup B)$ is given by $P(A \cup B) = P(A) + P(B)$

(4) $P(E) + P(\bar{E}) = 1.$

$$P(\bar{E}) = 1 - P(E)$$

$$P(E) = 1 - P(\bar{E})$$

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THEOREMS ON PROBABILITY (LAWS OF PROBABILITY)

There are two important Laws of Probability namely,

1. Addition Theorem on Probability
2. Multiplication Theorem.

*

Addition Theorem

If A and B are two events, then probability of happenning of the events A or B is denoted by $P(A \cup B)$ and is given by

$P(A \cap B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

if the two events A and B are mutually exclusive then $A \cap B = \emptyset$ $P(A \cap B) = 0$

$$\therefore \quad P(A \cup B) = P(A) + P(B)$$

Multiplication Theorem on Probability

Let A and B be two events. Then probability of happenning of the both the events A and B is denoted by $P(A \cap B)$ and is given by

$$P(A \cap B) = P(A) \cdot P(B/A)$$

where $P(B/A)$ is probability of happenning of the event B after happenning of the Event A.

- if Two events A and B are independent, the probability that both will occur is equal to the product of their individual probabilities $P(A \cap B) = P(A) \cdot P(B)$

- if A, B, C are independent events

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

CONDITIONAL PROBABILITY

let A and B be two events. The probability of happenning of the event B after happenning of the event A is called "Conditional Probability of B given A". It is denoted by $P(B/A)$

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margin

$$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \qquad P(A) \neq 0$$

قليل الأهمية أصغر
marginal

Note: If E_1 and E_2 are two events

$$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)} \qquad P(E_1) \neq 0$$

Marginal Probability:

Probability of a single event is called Marginal probability.

Joint Probability:

The probability of happening of two or more events together is called joint probability.

Example:

	Male (M)	Female (F)	
Doctor (D)	100	200	300
Nurse (N)	150	250	400
	250	450	700

Marginal Probabilities:

$$P(D) = \frac{300}{700}$$

$$P(N) = \frac{400}{700}$$

$$P(M) = \frac{250}{700}$$

$$P(F) = \frac{450}{700}$$

Joint Probabilities:

$$P(D \cap M) = \frac{100}{700}$$

$$P(D \cap F) = \frac{200}{700}$$

$$P(N \cap M) = \frac{150}{700}$$

$$P(N \cap F) = \frac{\cancel{400}\ 150}{700}$$

$$P\left(\frac{M}{D}\right) = \frac{100}{300} \qquad P\left(\frac{D}{M}\right) = \frac{100}{250}$$

$$P\left(\frac{F}{D}\right) = \frac{200}{300} \qquad P\left(\frac{N}{M}\right) = \frac{150}{250}$$

$$P\left(\frac{M}{N}\right) = \frac{150}{400} \qquad P\left(\frac{D}{F}\right) = \frac{200}{450}$$

$$P\left(\frac{F}{N}\right) = \frac{250}{400} \qquad P\left(\frac{N}{F}\right) = \frac{250}{450}$$

Conditional Probabilities

* Independent Events

Two events are said to be independent events, if happenning of one event does not affect the happenning of other event.

if E_1 and E_2 are independent events

then $P\left(\frac{E_2}{E_1}\right) = P(E_2)$

** Dependent events

Two events are said to be dependent events, if happenning one event affects the happenning of other event.

if E_1 and E_2 are dependent events

then $P\left(\frac{E_2}{E_1}\right) \neq P(E_2)$.

BAYE'S THEOREM

Let $A_1, A_2, A_3, \ldots\ldots A_n$ be 'n' mutually exclusive events, let 'B' be the event which will happen in conjunction with any one of $A_1, A_2, A_3, \ldots\ldots A_n$. Then

$$P\left(\frac{A_i}{B}\right) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i)\cdot P\left(\frac{B}{A_i}\right)}{P(B)} \qquad i = 1, 2, \ldots n$$

where

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \cdots + P(A_n \cap B)$$

$$= P(A_1)\cdot P\left(\frac{B}{A_1}\right) + P(A_2)\cdot P\left(\frac{B}{A_2}\right) + \cdots + P(A_n)\cdot P\left(\frac{B}{A_n}\right)$$

$P(A_1), P(A_2), \ldots P(A_n)$ are called Priori Probabilities.

$P\left(\frac{B}{A_1}\right), P\left(\frac{B}{A_2}\right), \ldots P\left(\frac{B}{A_n}\right)$ are called Conditional Probabilities

Correspond

Probability Distribution

We know in any random experiment there are several possible outcomes. Each outcome has a specific chance or probability. The set of outcomes with their corresponding probabilities is called probability distribution.

Ex:

event	Head	Tail
Probability	1/2	1/2

~~corresponding~~

it represents how the probability 1 is distributed among different outcomes of a Trial.

There are two types of probability distributions.

1. Discrete probability Distribution
2. Continuous probability distribution

Discrete Probability Distribution:

A probability distribution in which the variable takes a limited number of values is called discrete probability distribution.

Continuous Probability distribution

A probability distribution in which the variable takes any value with in a given range of values is called Continuous probability distribution.

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Random variable:

A function defined on a Sample space is Called a random variable.

Ex: when Coin Tossed

$S = \{H, T\}$

outcome	H	T
Prob	½	½

let X = no. of Tails $X = \{0, 1\}$

$X = x_i$	0	1
$P(X = x_i)$	½	½

Discrete Random variable:

A random variable which is allowed to take only a limited number of values is called discrete random variable.

Continuous Random variable

A random variable which is allowed to take any value with in a given range of values is called Continuous random variable.

Bernoulli Process

A process in which an experiment is repeated a finite number of times and each experiment is having only two possible outcomes such as Success & Failure, The prob of success is Constant and all the Trials are independent.

تجربة / اختبار trial

Binomial Distribution

This distribution is applied when

1. The number of Trials (تجارب) (n) is finite (محدود/منتهي)
2. There are only two possible outcomes like Head or Tail, Male or Female, True or False, etc (success or failure)
3. probability of one event (success) is Constant (ثابت)
4. All the Trials are independent

<u>Definition</u>: let $X = \{x_1, x_2, x_3 \dots x_n\}$ be a discrete random variable. Let p be the probability of success. If the probability of 'r' success is given by

$$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

Then the distribution is said to be following a Binomial Distribution.

(where n = number of trials and $q = 1 - p$.)